CALIBRATING A SIX-PORT REFLECTOMETER USING THREE KNOWN LOADS

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ABSTRACT

A technique for calibrating an arbitrary six-port network using nine unknown loads and three known loads is described. The technique is similar to the self-calibration technique first developed by Engen. However, it uses full six-port to four-port reduction so that it is quite insensitive to errors. The theory is illustrated with typical measurement results showing an accuracy of better than 1.0% achievable without optimization of the six-port to four-port reduction constants.

INTRODUCTION

In the last few years network analysers based on six-port networks have been proven a viable alternative to conventional network analysers using heterodyning systems. The six-port technique relates power measurements to the magnitude and phase of an unknown parameter (i.e. reflection coefficient). As vector detection has been replaced with scalar measurements more complex mathematical equations result and a desktop computer is used to perform the necessary calculations.

The key factor in achieving good measurement results using six-port techniques, is the accurate calibration of the six-port network. Methods available to calibrate the six-port network can be divided into two categories. One category requires many, (greater than five) accurately known standards [1]. Techniques of the second category require, in the first stage many (approx. nine) unknown but repeatable standards and finally fewer (three) accurately known standards. Procedures of the second category are of major interest as they are suited to automatic calibration of the six-port in a large frequency range.

Methods of the second category were first introduced by Engen [3]. He showed how the six-port network could be reduced to an equivalent four-port network from which vector detection was achieved. It was also shown that constraints on the six-port network could be described as a 3-dimensional surface. This surface was then related to the four-port parameters. However, parts of Engens' theory were based on five-port to four-port reduction. Neglecting the information given by the sixth detector produced some ambiguities which, even after imposing restrictions on the six-port network, could not be clearly resolved.

In the following paper an improved technique for calibrating the six-port reflectometer is presented. Full information of all power detector readings is exploited to overcome the ambiguities met in earlier theory [3].

CALIBRATION METHOD

Six-port to Four-port Reduction

The six-port reflectometer is governed by the following equations (1) :

$$\frac{A\Gamma+B}{G\Gamma+1} = \sqrt{\alpha}e^{j\phi}, \quad \frac{C\Gamma+D}{G\Gamma+1} = \sqrt{\beta}e^{j\psi}, \quad \frac{E\Gamma+F}{G\Gamma+1} = \sqrt{\gamma}e^{j\eta} \qquad (1)$$

where: α,β,γ are power ratios measured w.r.t the reference power at fourth network port, Γ is the reflection coefficient of the load, A,B,C,D,E,F,G are the unknown six-port constants and ϕ,ψ,η are fictitious power ratio angles.

The phases ϕ,ψ,η are unknown variables usually not included in conventional six-port circuit equations but are used here for convenience. Each of the equations in (1) represents a four-port equation similar in type to that used to describe a conventional network analyser. The reflection coefficient (Γ) and the phases ϕ,ψ,η can then be eliminated from (1) to give an expression which describes the full set of constraints governing the operation of the general six-port network.

$$\sum_{i=1}^{10} T_{ki} Z_i = 0 \qquad (2)$$

where: k is the measurement number

$$T_{k1}=\alpha^2,\ T_{k2}=\alpha,\ T_{k3}=\beta^2,\ T_{k4}=\gamma^2,\ T_{k5}=\beta\gamma$$

$$T_{k6}=\alpha\beta,\ T_{k7}=\beta,\ T_{k8}=\alpha\gamma,\ T_{k9}=\gamma,\ T_{k10}=1$$

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The Zi's are constants related to the primary parameters A,B,C,D,E,F,G.

$$Z_1 = (X/Y)^2+(P/Q)^2-2(XP)/(YQ)\cos(\xi-\tau)$$

$$Z_2 = 4\cos^2(\xi-\tau)-2\cos(\xi-\tau)[(XQ)/(YP)+(YP)/(XQ)]$$

$$Z_3 = 1/(XY)^2 \quad ; \quad Z_4 = 1/(PQ)^2$$

$$Z_5 = -[2/(XYPQ)\cos(\xi-\tau)]$$

$$Z_6 = -[2/Y^2-2P/(QXY)\cos(\xi-\tau)]$$

$$Z_7 = -[2/X^2-2Q/(PXY)\cos(\xi-\tau)]$$

$$Z_8 = -[2/Q^2-2X/(YPQ)\cos(\xi-\tau)]$$

$$Z_9 = -[2/P^2-2Y/(XPQ)\cos(\xi-\tau)]$$

$$Z_{10} = (Y/X)^2+(Q/P)^2-2(YQ)/(XP)\cos(\xi-\tau)$$

where : $X=|x|$, $Y=|y|$, $P=|p|$, $Q=|q|$

$$x^*y = XYe^{j\tau} \quad , \quad p^*q = PQe^{j\xi} \quad ; \; *\text{- conjugate}$$

$$x = (DG-C)/(BG-A) \quad , \quad y = (BC-DA)/(BG-A)$$

$$p = (FG-E)/(BG-A) \quad , \quad q = (BE-FA)/(BG-A)$$

The final equation determines the six-port to four-port reduction where $X,Y,P,Q,\cos(\xi-\tau)$ are the unknown reduction parameters. Observation shows the equation is symmetrical in terms of the power ratios α,β,γ. To find $X,Y,P,Q,\cos(\xi-\tau)$ a similar procedure to that suggested in [3] can be used. Taking nine measurements of the power ratios α,β,γ nine equations of the form (2) are obtained. The resulting set of equations can then be written in matrix form (3).

$$[T_{ki}][t_i] = -1 \qquad k,i=1,2,..9 \tag{3}$$

The ti's are defined as the Zi's in (2) but have been normalized versus Z_{10} : $t_i=Z_i/Z_{10}$. Equation (3) represents a system of linear equations and can be solved using standard methods. The five constants $X,Y,P,Q,\cos(\xi-\tau)$ are then determined from the following procedure :

$$\cos(\xi-\tau) = -t_5 . s/(2\sqrt{t_3 t_4}) \quad , \quad s = \text{sign}(t_3)$$

$$P/Q = \frac{t_1(1+1/V-2\cos(\xi-\tau)V^{-1/2})^{1/4}}{(1+V-2\cos(\xi-\tau)V^{1/2})} \tag{4}$$

$$X/Y = P/Q\sqrt{V} \; , \; XY = [t_3 Z_{10}]^{-1/2} \; , \; PQ = [t_4 Z_{10}]^{-1/2}$$

$$X = (XY.X/Y)^{1/2} \; ; \; P = (PQ.P/Q)^{1/2}$$

$$Y = (XY.Y/X)^{1/2} \; ; \; Q = (PQ.Q/P)^{1/2}$$

In practice $X,Y,P,Q,\cos(\xi-\tau)$ will only be determined approximately due to the finite precision with which the power ratios are measured. Improved values maybe found if more than nine measurements are taken and the system (3) solved in a least squares sense.

Alternatively, using the values calculated as a first approximation, a nonlinear optimization based on (2) may be used to generate more accurate results [3].

The next step is to determine the fictitious phases ϕ or ψ or η. Actually we need only determine the values to within a constant deviation. The procedure is the same for any of the three angles and is only illustrated here for the angle ϕ.

Two equations relating the phase angle ϕ to reduction constants can be found from (1).

$$\cos(\phi-\tau) = \frac{\beta-\alpha X^2+Y^2}{2XY\sqrt{\alpha}} \tag{5}$$

$$\tan(\phi-\tau) = \frac{1}{\tan(\xi-\tau)} \cdot \left[-1+\frac{1}{\cos(\xi-\tau)} \cdot \frac{XY}{PQ} \cdot \frac{(\gamma-\alpha P^2-Q^2)}{(\beta-\alpha X^2-Y^2)}\right] \tag{6}$$

Equation (5) is based on five-port to four-port reduction relationships as one of the power ratios is missing in this expression. Equation (6) however, uses full six-port to four-port reduction. The equation shown in (5) may be related to that used in Engen's theory.

The simultaneous use of both equations (5) and (6) will give a unique value for the desired phase, $\phi-\tau$. The only remaining ambiguity in using equations (5),(6) results from not knowing the sign of $\tan(\xi-\tau)$. However, once the sign is chosen a unique sequence of $\phi-\tau$ angles are obtained. If later tests prove the sign choice to be wrong, the calculated sequence $\phi-\tau$ has only to be replaced by their negative values to obtain correct results.

The phase sequences for $\psi-\tau_1$ and $\eta-\tau_2$ may be obtained in a similar manner as described above. However, it should be noted that equation (2) is symmetrical in terms of α,β,γ and so versus ϕ,ψ,η. Thus, by rearranging the coefficients of (2) new values of X_1, Y_1, P_1, Q_1, $\cos(\xi_1-\tau_1)$ and X_2, Y_2, P_2, Q_2, $\cos(\xi_2-\tau_2)$ can be obtained through (4) without having to resolve (3).

Four-Port Constant Determination

The final part of the calibration requires the calculation of the parameters for any of the three, four-ports described by equations (1). This is achieved through the use of three known calibration loads: Γ_1, Γ_2, Γ_3. The three resulting complex equations in three complex constants may then be obtained as:

$$(A\Gamma_i+B)/(G\Gamma_i+1) = \sqrt{\alpha}e^{j(\phi_i-\tau)} \qquad i=1,2,3. \tag{7}$$

The existence of τ in (7) does not contradict equation (1) since only the difference between ϕ's are of importance. Solving equation (7) will then yield the desired four-port parameters. i.e. A,B,G. The four-port parameters of the two remaining expressions in (1) are calculated in a similar fashion.

As each of the four-port equations is capable of independent measurement of the unknown quantity (Γ), it appears reasonable to find only one set of four-port constants. This can be done if the user is willing to tradeoff accuracy for speed of operation.

REFLECTION COEFFICIENT MEASUREMENTS

To verify the new calibration scheme measured and predicted reflection coefficients of a number of loads were compared.

The six-port tested was a design similar to that first described by Engen [2]. It was constructed of Q,H hybrids made in stripline technology and terminated in WG-16 rectangular waveguide. For this configuration the separation of the q's [2] approach 120°. Tests were carried out in the 8-12 GHz range. Six-port to four-port reduction was achieved by generating nine different reflection coefficients using an attenuator backed by a sliding short. The values chosen were : four positions of the sliding short, phase difference greater than 45°, four similar positions with a constant attenuation (1-3 dB) and an imperfect match realized by setting the attenuator to 20dB. The three known standards used in calibration were the open, short and offset short (Phase offset 45-135°).

Table 1 shows the measured reflection coefficients of different positions of a sliding termination. The termination used consisted of a 1dB attenuator backed by a sliding short circuit. The operating frequency was 10 GHz and no optimization of the six-port to four-port reduction constants was used.

PREDICTED Γ		MEASURED Γ	
|Γ|	PHASE Γ	|Γ|	PHASE Γ
0.5122	-145.2	0.5107	-145.2
0.5122	169.8	0.5077	169.3
0.5122	124.8	0.5083	124.6
0.5122	79.8	0.5087	79.4
0.5122	34.8	0.516	34.1
0.5122	-10.2	0.516	-10.79
0.5122	-55.2	0.5135	-55.69
0.5122	-100.2	0.5168	-101.1

Table 1 : Comparison of Measured and Predicted Reflection Coefficients of Sliding Termination

The important features to note are that $|\Gamma|$ had to be constant ($|\Gamma|_{av} = 0.5122$) and the phase separation should be 45°. The observed worst case errors are therefore, 0.9% in the magnitude and 0.9° in the phase.

Table 2 shows measured and predicted reflection coefficients on the unit circle under the same operating conditions.

The worst case errors are again of the same order as shown previously. Results from tests at other frequencies in the 8-12 GHz range showed that the accuracy was essentially independent of frequency.

PREDICTED Γ		MEASURED Γ	
|Γ|	PHASE Γ	|Γ|	PHASE Γ
1.0	0.0	1.00015	0.051
1.0	45.0	0.9975	44.63
1.0	90.0	0.99956	90.04
1.0	135.0	0.9891	135.25
1.0	180.0	1.0006	180.04
1.0	-135.0	0.993	-134.0
1.0	-90.0	0.9977	-90.5
1.0	-45.0	1.0001	-45.16

Table 2 : Comparison of Measured and Predicted Reflection Coefficients of Sliding Short

CONCLUSIONS

The calibration procedure described is a refined self-calibration technique for the six-port reflectometer. The new technique retaines the advantages of earlier methods, using a minimum of accurately known loads but without the necessity for hardware restrictions inherent in other theory [3]. The new procedure boasts full six-port to four-port reduction theory in which all ambiguities are eliminated and network symmetry is fully exploited.

Accuracy is very good, showing worst case errors of less than 1% are achievable without any optimization of the reduction constants. Calibration is also relatively fast, taking about as long as calibration techniques which use many known calibration standards.

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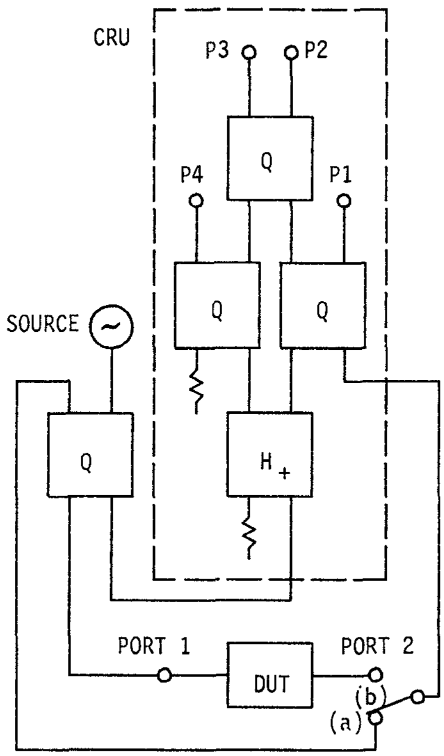

Fig.1: Six-port Reflectometer Configuration
Q,H - 90° and 180° hybrids.
CRU - Complex Ratio Measuring Unit.
Switch position (a) Reflective mode
(b) Transmittive mode

* The measurement system shown in Fig. 1 is analogous to the conventional Network Analyser (NA) system. However, the conventional hetrodyning circuitry used within the CRU has been replaced with simple power detection. Full analogy with the old type of NA operations is maintained through use of the six-port to four-port reduction described earlier. The necessary reduction has been demonstrated for the device operating in the reflective mode but may also be achieved in the transmittive mode. It has been experimentally verified, that irrespective of the choice of operating mode, the reduction will be identical. To within the bounds of the measurement error the same reduction constants are calculated using either a reflective or transmittive mode reduction.